

Is it possible to use the same sort of simple reversal of operations to find the inverse of this function? If so, do it. If not, explain what is different about this function that makes this task difficult.

82. The Identity Function The function $I(x) = x$ is called the **identity function**. Show that for any function f we have $f \circ I = f$, $I \circ f = f$, and $f \circ f^{-1} = f^{-1} \circ f = I$. (This means that the identity function I behaves for functions and composition just like the number 1 behaves for real numbers and multiplication.)

83. Solving an Equation for an Unknown Function In Exercise 65 of Section 2.7 you were asked to solve equations in which the unknowns were functions. Now that we know about inverses and the identity function (see Exercise 82), we can use algebra to solve such equations. For

instance, to solve $f \circ g = h$ for the unknown function f , we perform the following steps:

$$\begin{array}{ll} f \circ g = h & \text{Problem: solve for } f \\ f \circ g \circ g^{-1} = h \circ g^{-1} & \text{Compose with } g^{-1} \text{ on the right} \\ f \circ I = h \circ g^{-1} & g \circ g^{-1} = I \\ f = h \circ g^{-1} & f \circ I = f \end{array}$$

So the solution is $f = h \circ g^{-1}$. Use this technique to solve the equation $f \circ g = h$ for the indicated unknown function.

- Solve for f , where $g(x) = 2x + 1$ and $h(x) = 4x^2 + 4x + 7$
- Solve for g , where $f(x) = 3x + 5$ and $h(x) = 3x^2 + 3x + 2$

2 Review

Check

Define each concept in your own words. (Check by referring to the definition in the text.)

- Function
- Domain and range of a function
- Graph of a function
- Independent and dependent variables

Give an example of each type of function.

- Constant function
- Linear function
- Quadratic function

Sketch by hand, on the same axes, the graphs of the following functions.

$$\begin{array}{ll} f(x) = x & (b) \ g(x) = x^2 \\ h(x) = x^3 & (d) \ j(x) = x^4 \end{array}$$

State the Vertical Line Test.

State the Horizontal Line Test.

What is the average rate of change of the function f between points defined?

Define each concept in your own words.

- g function
- g function
- constant function

7. Suppose the graph of f is given. Write an equation for each graph that is obtained from the graph of f as follows.

- Shift 3 units upward
- Shift 3 units downward
- Shift 3 units to the right
- Shift 3 units to the left
- Reflect in the x -axis
- Reflect in the y -axis
- Stretch vertically by a factor of 3
- Shrink vertically by a factor of $\frac{1}{3}$
- Stretch horizontally by a factor of 2
- Shrink horizontally by a factor of $\frac{1}{2}$

- What is an even function? What symmetry does its graph possess? Give an example of an even function.
- What is an odd function? What symmetry does its graph possess? Give an example of an odd function.

9. Write the standard form of a quadratic function.

10. What does it mean to say that $f(3)$ is a local maximum value of f ?

11. Suppose that f has domain A and g has domain B .

- What is the domain of $f + g$?
- What is the domain of fg ?
- What is the domain of f/g ?

12. How is the composite function $f \circ g$ defined?

13. (a) What is a one-to-one function?

(b) How can you tell from the graph of a function whether it is one-to-one?

(c) Suppose f is a one-to-one function with domain A and

range B . How is the inverse function f^{-1} defined? What is the domain of f^{-1} ? What is the range of f^{-1} ?

(d) If you are given a formula for f , how do you find a formula for f^{-1} ?

(e) If you are given the graph of f , how do you find the graph of f^{-1} ?

Exercises

1. If $f(x) = x^2 - 4x + 6$, find $f(0)$, $f(2)$, $f(-2)$, $f(a)$, $f(-a)$, $f(x+1)$, $f(2x)$, and $2f(x) - 2$.

2. If $f(x) = 4 - \sqrt{3x - 6}$, find $f(5)$, $f(9)$, $f(a+2)$, $f(-x)$, $f(x^2)$, and $[f(x)]^2$.

3. The graph of a function f is given.

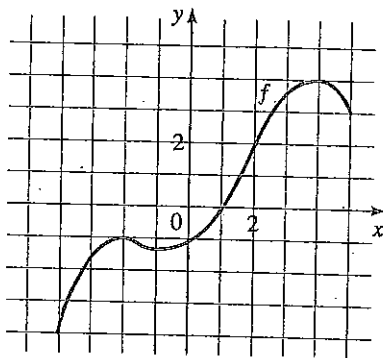
(a) Find $f(-2)$ and $f(2)$.

(b) Find the domain of f .

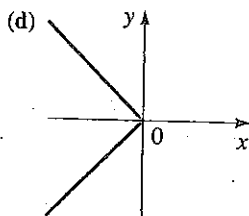
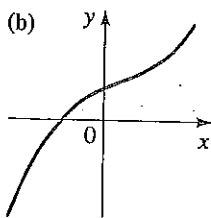
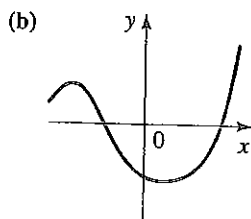
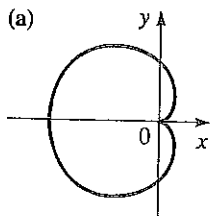
(c) Find the range of f .

(d) On what intervals is f increasing? On what intervals is f decreasing?

(e) Is f one-to-one?



4. Which of the following figures are graphs of functions? Which of the functions are one-to-one?



5-6 \square Find the domain and range of the function.

5. $f(x) = \sqrt{x+3}$

6. $F(t) = t^2 + 2t + 5$

7-14 \square Find the domain of the function.

7. $f(x) = 7x + 15$

8. $f(x) = \frac{2x+1}{2x-1}$

9. $f(x) = \sqrt{x+4}$

10. $f(x) = 3x - \frac{2}{\sqrt{x+1}}$

11. $f(x) = \frac{1}{x} + \frac{1}{x+1} + \frac{1}{x+2}$

12. $g(x) = \frac{2x^2+5x+3}{2x^2-5x-3}$

13. $h(x) = \sqrt{4-x} + \sqrt{x^2-1}$

14. $f(x) = \frac{\sqrt[3]{2x+1}}{\sqrt[3]{2x}+2}$

15-32 \square Sketch the graph of the function.

15. $f(x) = 1 - 2x$

16. $f(x) = \frac{1}{3}(x-5)$, $2 \leq x \leq 8$

17. $f(t) = 1 - \frac{1}{2}t^2$

18. $g(t) = t^2 - 2t$

19. $f(x) = x^2 - 6x + 6$

20. $f(x) = 3 - 8x - 2x^2$

21. $g(x) = 1 - \sqrt{x}$

22. $g(x) = -|x|$

23. $h(x) = \frac{1}{2}x^3$

24. $h(x) = \sqrt{x+3}$

25. $h(x) = \sqrt[3]{x}$

26. $H(x) = x^3 - 3x^2$

27. $g(x) = \frac{1}{x^2}$

28. $G(x) = \frac{1}{(x-3)^2}$

29. $f(x) = \begin{cases} 1-x & \text{if } x < 0 \\ 1 & \text{if } x \geq 0 \end{cases}$

30. $f(x) = \begin{cases} 1-2x & \text{if } x \leq 0 \\ 2x-1 & \text{if } x > 0 \end{cases}$

31. $f(x) = \begin{cases} x+6 & \text{if } x < -2 \\ x^2 & \text{if } x \geq -2 \end{cases}$

32. $f(x) = \begin{cases} -x & \text{if } x < 0 \\ x^2 & \text{if } 0 \leq x < 2 \\ 1 & \text{if } x \geq 2 \end{cases}$



33. Determine which viewing rectangle produces the most appropriate graph of the function $f(x) = 6x^3 - 15x^2 + 4x - 1$.

(i) $[-2, 2]$ by $[-2, 2]$

(ii) $[-8, 8]$ by $[-8, 8]$

(iii) $[-4, 4]$ by $[-12, 12]$

(iv) $[-100, 100]$ by $[-100, 100]$

34. Determine which viewing rectangle produces the most appropriate graph of the function $f(x) = \sqrt{100 - x^2}$.

- (i) $[-4, 4]$ by $[-4, 4]$
- (ii) $[-10, 10]$ by $[-10, 10]$
- (iii) $[-10, 10]$ by $[-10, 10]$
- (iv) $[-100, 100]$ by $[-10, 100]$

35–38 □ Draw the graph of the function in an appropriate viewing rectangle.

35. $f(x) = x^2 + 25x + 18$

36. $f(x) = 1.1x^3 - 9.6x^2 - 1.4x + 3.2$

37. $f(x) = \frac{x}{\sqrt{x^2 + 16}}$

38. $f(x) = |x(x + 2)(x - 4)|$

39. Find, approximately, the domain of the function $f(x) = \sqrt{x^3 - 4x}$.

40. Find, approximately, the range of the function $f(x) = x^4 - x^3 + x^2 - 3x - 6$.

41–44 □ Find the average rate of change of the function between the given points.

41. $f(x) = x^2 + 3x$; $x = 0, x = 2$

42. $f(x) = \frac{1}{x - 2}$; $x = 1, x = 8$

43. $f(x) = \frac{1}{x}$; $x = 3, x = 3 + h$

44. $f(x) = (x + 1)^2$; $x = a, x = a + h$

45–46 □ Draw a graph of the function f , and determine the intervals on which f is increasing and on which f is decreasing.

45. $f(x) = x^3 - 4x^2$

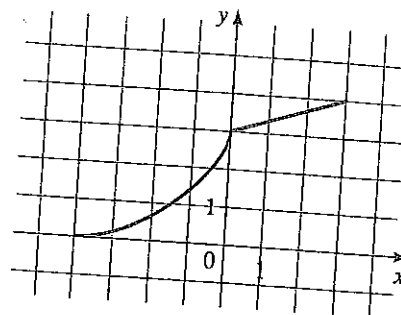
46. $f(x) = |x^4 - 16|$

47. Suppose the graph of f is given. Describe how the graphs of the following functions can be obtained from the graph of f .

- (a) $y = f(x) + 8$
- (b) $y = f(x + 8)$
- (c) $y = 1 + 2f(x)$
- (d) $y = f(x - 2) - 2$
- (e) $y = f(-x)$
- (f) $y = -f(-x)$
- (g) $y = -f(x)$
- (h) $y = f^{-1}(x)$

48. The graph of f is given. Draw the graphs of the following functions.

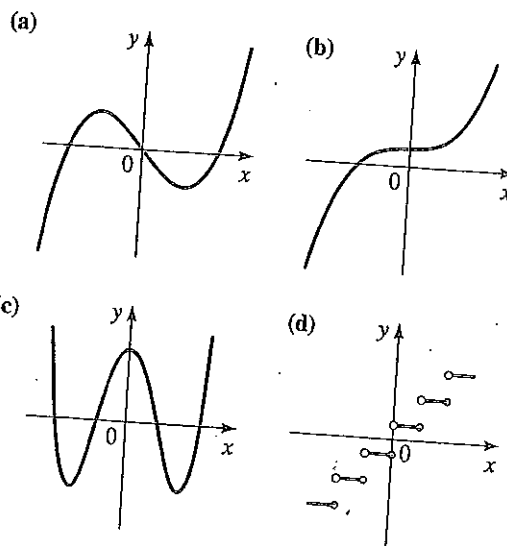
- (a) $y = f(x - 2)$
- (b) $y = -f(x)$
- (c) $y = 3 - f(x)$
- (d) $y = \frac{1}{2}f(x) - 1$
- (e) $y = f^{-1}(x)$
- (f) $y = f(-x)$



49. Determine whether f is even, odd, or neither.

- (a) $f(x) = 2x^5 - 3x^2 + 2$
- (b) $f(x) = x^3 - x^7$
- (c) $f(x) = \frac{1 - x^2}{1 + x^2}$
- (d) $f(x) = \frac{1}{x + 2}$

50. Determine whether the function in the figure is even, odd, or neither.



- 51. Express the quadratic function $f(x) = x^2 + 4x + 1$ in standard form.
- 52. Express the quadratic function $f(x) = -2x^2 + 12x + 12$ in standard form.
- 53. Find the minimum value of the function $g(x) = 2x^2 + 4x - 5$.
- 54. Find the maximum value of the function $f(x) = 1 - x - x^2$.

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Combining Functions

Given the following pairs of functions, find, in simplest terms;

a. $f(x) + g(x)$ b. $f(x) - g(x)$ c. $f(x) \cdot g(x)$ d. $\frac{f(x)}{g(x)}$

1. $f(x) = x^2 - 2x$; $g(x) = 2x^2 + 5x - 3$

2. $f(x) = 2x + 5$; $g(x) = 1 - \cos x$

3. (remember, improper fractions and horizontal fraction bars)

x	-3	-2	-1	0	1	2	3
$f(x)$	7	4	-2	3	-3	5	3
$g(x)$	8	-3	9	2	-1	3	2
$f(x) + g(x)$							
$f(x) - g(x)$							
$f(x) \cdot g(x)$							
$\frac{f(x)}{g(x)}$							

4. On a separate piece of graph paper, graph the answers.

